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# Study of a Nonstationary Separation Method with Gas Centrifuge Cascade

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## ABSTRACT

A nonconventional gas centrifuge cascade, called the NFSW (no feed and single withdrawal) cascade, is studied in the separation of middle components by means of numerical simulation. The cascade has no feed and only a single withdrawal at either end of the cascade, different from conventional cascades, which usually have two withdrawals at the two ends of the cascade and one feed in between. The material to be separated is loaded in a reservoir at either end, and the desired component is enriched in either the reservoir or the withdrawal at the other end. The effects of the unit separation factor (equivalently, the cascade length) and the ratio of the upstreaming flow rate to the withdrawal rate are investigated on separation. The separation performance is evaluated in terms of the material recovery and the operation time efficiency, and is

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compared with those of the corresponding conventional cascades and another type of nonconventional cascade, the SW (single withdrawal) cascade. It is found that the NFSW cascade is superior to the conventional cascade and comparable in the material recovery with, but advantageous in the operation time efficiency over the SW cascade.

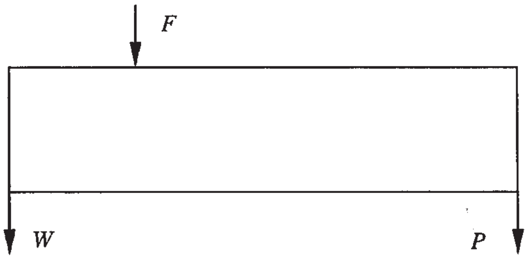
*Key Words:* Multicomponent isotope separation; Gas centrifuge cascade; Nonstationary cascade; Numerical simulation.

## INTRODUCTION

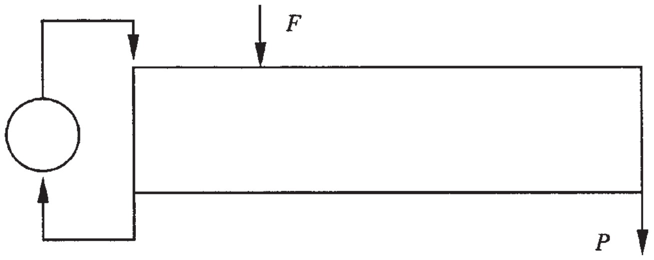
In the separation of multicomponent isotopic mixtures with gas centrifuge cascades operating in a conventional way (cf. Fig. 1a), enriching a middle component to a very high concentration is always much more difficult than an end component. This is particularly true when the length of cascade is not sufficiently long; a middle component remains a middle component even after several successive separation runs. With a sufficiently long cascade, only two runs suffice to enrich a middle component to any required concentration, but, as short a cascade as possible is always wanted from the consideration of practical factors, e.g., operation costs.

To achieve better separation performance in separating either an end component of small abundance or a middle component, the ideas of nonconventional methods are developed.[1] In these methods, separation is carried out by making use of transient processes, in which the concentration distributions along cascades change with time during the whole separation process. Such methods are also referred to as nonstationary methods. Figure 1 shows, schematically, the conventional method and two nonstationary methods. In Fig. 1b, the cascade has only one withdrawal, either product or waste withdrawal, so it is called the single withdrawal (SW) method. For an SW method, it has been shown in Zeng and Ying[2] that it is a very efficient method to separate end components, especially end components of small abundance. But it is not so advantageous over the conventional method in separating middle components, although it can be easily tuned to give a higher separation performance. Therefore, the SW method is not recommended for separating middle components, and development and investigation of new nonstationary separation methods are necessary.

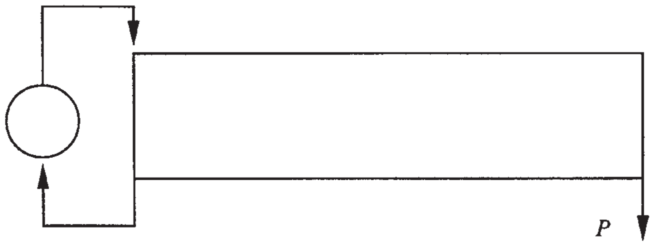
Another nonstationary method of separating middle components is proposed in Sosnin and Tcheltsov,[3] as illustrated in Fig. 1c, and has been successfully used to enrich the $^{123}$Te isotope to a high concentration, over 99.9%, as the target material for the production of the radionuclide $^{123}$I. For ease of reference, this method is referred to as the no feed and single withdra-



a. A conventional cascade



b. An SW (Single Withdrawal) cascade: either the product withdrawal or the waste withdrawal is used and a reservoir is equipped at the recirculating pipe on the closed end of the cascade



c. An NFSW (No Feed and Single Withdrawal) cascade: just like the SW cascade except for no feed

***Figure 1.*** Illustration of three types of cascades.

wal (NFSW) cascade or method. The question is: under what conditions and to what extent is the NFSW method suitable for separating middle components?

To answer this question, perhaps partially, is the main objective of this paper. The analyzing tool is a numerical simulation by using a second-order time-accurate method.[4] As usual, the analyses are carried out within the framework of comparing the following two indices of a cascade running in the NFSW mode with those in the conventional mode: the material recovery and the operation time efficiency (cf. Zeng and Ying[2]), for producing the same amount of the required component.

## The Partial Differential-Difference Equations and Solution

A more detailed illustration of an NFSW cascade with a withdrawal at the product end is given in Fig. 2. Assume that there are $N$ stages in the cascade, and the multicomponent isotopic mixture in consideration consists of $N_c$ components, which are numbered sequentially from the lightest to the heaviest. $P_N = P$ is the only withdrawal. At stage $n$, $G_n$, $L'_n$, and $L''_n$, are the entering flow, the head and the tail flows, respectively, in which the concentrations of the $i$th component are $C_{i,n}$, $C'_{i,n}$ and $C''_{i,n}$, respectively. At stage 1, the tail flow is recirculated, and, on the recirculation pipe, a reservoir with holdup $H''_1$ is mounted. Perhaps it is convenient to call the amount of the process gas filled in the reservoir as the load. $L''_1$ and $L''^{a}_1$ are the tail flows entering and leaving the reservoir, respectively, and the concentrations in the two flows are $C_1''$ and $C_1''^{a}$, respectively.

When the cascade starts to operate, two types of transition processes occur, one is the flow transition and another is the concentration transition.

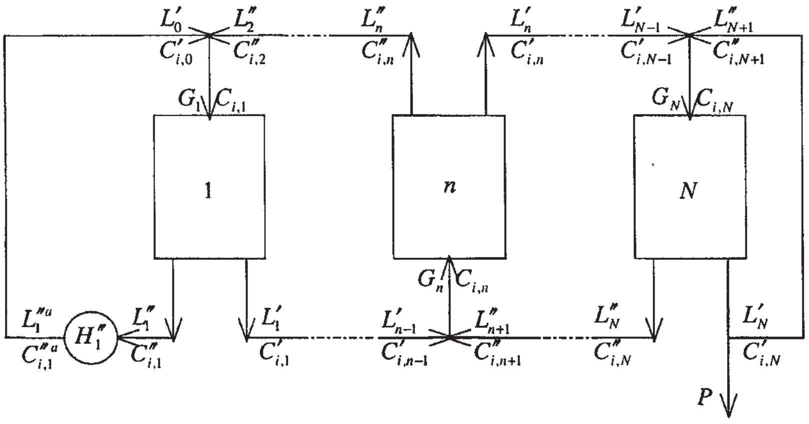


***Figure 2.*** A detailed illustration of an NFSW cascade with a withdrawal at the product end.

A flow transition process is one during which the hydraulic status, represented by all stage cuts and flow rates, in a cascade from one steady state changes to another steady state. Note that, in this process, the concentration distribution changes as well. A concentration transition process is one during which the concentration distribution changes from one steady state to another steady state. Note that even if the hydraulic status is steady, the concentration transition may still occur. Normally, a flow transition is much faster than a concentration transition; therefore, it is ignored in our study here for the sake of simplicity of analysis. That is, it is assumed that all flows in a cascade are constant. Because the holdup in the reservoir is designed to be much larger than the holdups in other places, for simplicity, the holdups in centrifuges and in the pipes connecting stages are not taken into account, except in the reservoir. Considering mass conservation of the *i*th component in the centrifuges at the *n*th stage gives

$$L'_{n-1}C'_{i,n-1} + L''_{n+1}C''_{i,n+1} - L''_{n}C'_{i,n} - L''_{n}C''_{i,n} = 0 \quad n = 1, 2, \ldots, N-1 \tag{1}$$

$$L'_{N-1}C'_{i,N-1} + L''_{N+1}C''_{i,N+1} - (L'_{N} + P)C'_{i,N} - L''_{N}C''_{i,N} = 0 \tag{2}$$

At the product end of the cascade, clearly

$$L''_{N+1} = L'_{N}, \quad C''_{i,N+1} = C'_{i,N} \tag{3}$$

At the waste end,

$$\frac{\partial H''_1(t)}{\partial t} = L''_1 - L'_0 = -P, \frac{\partial H''_1 \hat{C}''_{i,1}}{\partial t} = L''_1 C''_{i,1} - L'_0 C'_{i,0} \tag{4}$$

where $\hat{C}''_{i,1}$ is the averaged concentration of the *i*th component in the reservoir and approximated here by $\hat{C}''_{i1} \approx C'_{i,0}$. The cascade can also operate in another alternative way, i.e., instead of mounting a reservoir $H''_1$ at the waste end and having a withdrawal $P_N$ at the product end, a reservoir $H'_N$ at the product end and the waste withdrawal $W_1$ can be used. In this case, Eq. (3) should be replaced by

$$\frac{\partial H'_N(t)}{\partial t} = L'_N - L''_{N+1} = -W, \frac{\partial H'_N \hat{C}_{i,N}}{\partial t} = L'_N C'_{i,1} - L''_{N+1} C''_{i,N+1}$$

at the product end and Eq. (4) by

$$L'_0 = L''_1, \quad C'_{i,0} = C''_{i,1}$$

at the waste end. The separation characteristic of a stage is described by the following empirical formula[5,6]

$$\frac{C'_{i,n}/C''_{i,n}}{C'_{j,n}/C''_{j,n}} = \gamma_0^{M_j - M_i} \tag{5}$$

where $\gamma_0$ is the unit separation factor, and $M_i$ and $M_j$ are the molar weights of the $i$th and $j$th components, respectively. Equations (1)–(5) are the partial differential-difference equation system describing the transient behavior of concentration in the cascade. The following restriction condition

$$\sum_{i=1}^{N_c} C_{i,n} = \sum_{i=1}^{N_c} C'_{i,n} = \sum_{i=1}^{N_c} C''_{i,n} = 1 \tag{6}$$

applies to the concentrations at each stage. In the above equations, only the variables $C$, $C'$, and $C''$ are unknowns; the other quantities are specified.

Following Zeng and Ying,[7] Eq. (4) is discretized by using the second-order time-accurate unconditionally stable Crank-Nicolson scheme. The resulting algebraic equations, together with Eqs. (1), (2), (3), (5), and (6) consist of a nonlinear algebraic equation system and are easily solved by the so-called Q-iteration method.[8] At each time step, the solution accuracy criterion is

$$\max\left(\left|\sum_n C_{i,n} - 1\right|, \left|\sum_n C'_{i,n} - 1\right|, \left|\sum_n C''_{i,n} - 1\right|\right) \leq \varepsilon \tag{7}$$

where $\varepsilon$ is a small number. When condition Eq. (7) is satisfied, the solution marches to the next time step. Based on numerical tests, the number $10^{-6}$ is chosen for $\varepsilon$ and seems to be an appropriate value to give a sufficient solution accuracy, while keeping the number of iterations small enough.

## The NFSW Separation Process

To understand how an NFSW cascade works, the following experiment is carried out. The process gas is stored in the reservoir $H''_1$ initially. When the NFSW cascade is operating, the process gas in the reservoir is fed into the centrifuge cascade through $L'_0$ and, at the mean time, the process gas separated flows back into the reservoir through $L''_1$; $L'_0$ and $L'_1$ are adjusted so that the flow in the cascade maintains steady until the process gas in the reservoir is evacuated. As the gas travels through the cascade of centrifuges to the withdrawal end, one can imagine that the lightest component reaches the end first and then the next heavier component, and then the next. So in this way, at the withdrawal $P$, the components of different molar weights can be withdrawn successively. Figure 3 shows the behavior of the concentrations vs. time in $P$ for the 9 Xenon isotopes obtained by using a cascade with $\gamma_0 = 1.4$, $N = 11$, $P = 1$, $L'_n = 10$ ($n = 1, 2, \ldots, N$), and the initial amount of the process gas filled in the reservoir (the initial load), denoted as $H_1''^{(0)}$ here, $H_1''^{(0)} = 100$. The initial condition for the concentration in the cascade is a constant concentration distribution, given in Table 1. From Fig. 3, we notice that the concentrations at $t = 0$ do not correspond to the initial values given in Table 1. Actually, at $t = 0$, the concentrations are the values as given by Table 1, which cannot be clearly seen because they are plotted right on the ordinate axis. But at the first time step, the concentrations jump to the values as seen in Fig. 3. This is because the flow transition process is ignored and Eq. (5) is used, which holds for steady state. Consequently, the concentration transition cannot be revealed and incur a sudden change. The same happens to Figs. 4–7. Here the dimensions of the dimensional quantities are not specified for simplicity. But it should be kept in mind that the quantities of the same type should have the same dimension, for example, if the dimension for mass is kg and that for time is hr, then the flow rates should have the dimension kg/hr. That the concentrations of the components for the initial condition are so chosen is meant to demonstrate the separation phenomena of the NFSW cascade more clearly. Indeed, it can be seen from

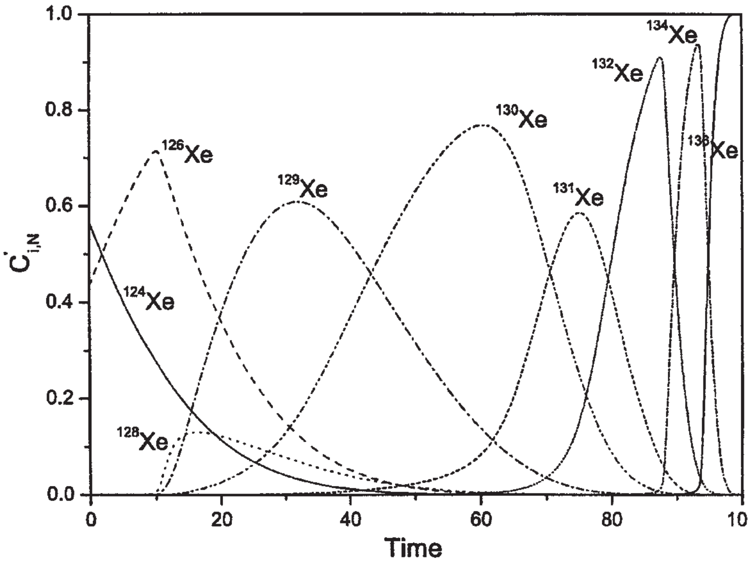


***Figure 3.*** The concentration vs. time in $P$ for $\gamma_0 = 1.4$, $P = 1$, $L'_n = 10$.

***Table 1.*** The initial concentrations of the nine Xenon components.

| Components | Concentration |
|---|---|
| $^{124}$Xe | 0.07 |
| $^{126}$Xe | 0.15 |
| $^{128}$Xe | 0.03 |
| $^{129}$Xe | 0.2 |
| $^{130}$Xe | 0.25 |
| $^{131}$Xe | 0.1 |
| $^{132}$Xe | 0.1 |
| $^{134}$Xe | 0.05 |
| $^{136}$Xe | 0.05 |

Fig. 3 that to enrich a particular component, one can just collect the withdrawal at a particular time interval; for instance, collecting *P* from time 10 to 60 to enrich $^{128}$Xe. Although the initial concentrations of all components are not large, the peak concentration of any component has significant elevation in *P*. Therefore, making better use of this nonstationary process may obtain a high separation performance.

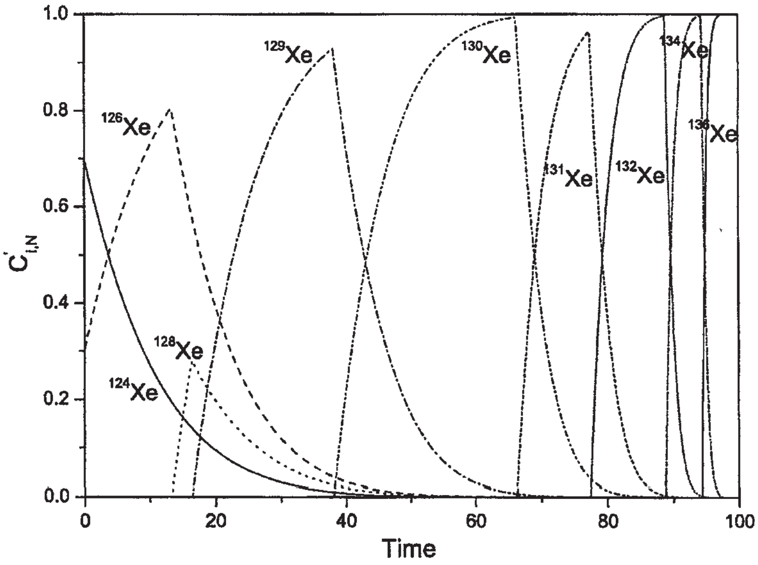


***Figure 4.*** The concentration vs. time in *P* for $\gamma_0 = 5$, $P = 1$, $L'_n = 10$.

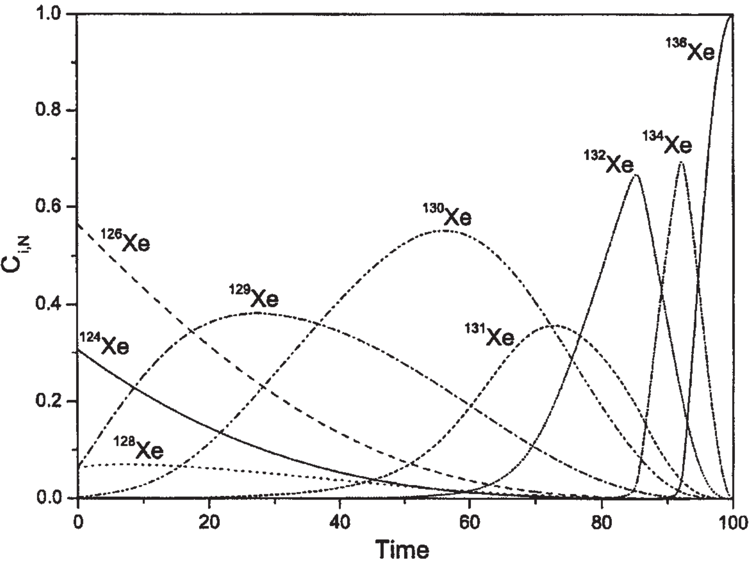


***Figure 5.*** The concentration vs. time in *P* for $\gamma_0 = 1.4$, P = 1, $L'_n = 5$.

However, we also observe from Fig. 3 that the curves for different components are intertwined and some of them span cross rather wide ranges, which leads to low resolution. By resolution, here we mean how well a component is separated from its neighboring components in a plot, e.g., Fig. 3, of concentration vs. time at the withdrawal point. We think it suffices to use this qualitative concept of resolution and so do not pursue a quantitative definition. At a low resolution, when collecting one component, some fractions of the neighboring components are collected at the same time. When the components are separated completely from each other, the highest resolution is achieved. In this case, the concentration-time plot of a component would be like a square step function, which we will see later. It is natural to think that a larger separation factor would enhance the resolution, and so the effect of the separation factor on the resolution is discussed now.

## The Effect of the Separation Factor $\gamma_0$ and Cascade Length *N*

Figure 4 is plotted with the results obtained by using the same data as for Fig. 3 except that the separation factor is 5. It is clearly seen that for increasing $\gamma_0$, the profiles of the curves are sharpened, and the concentration span of

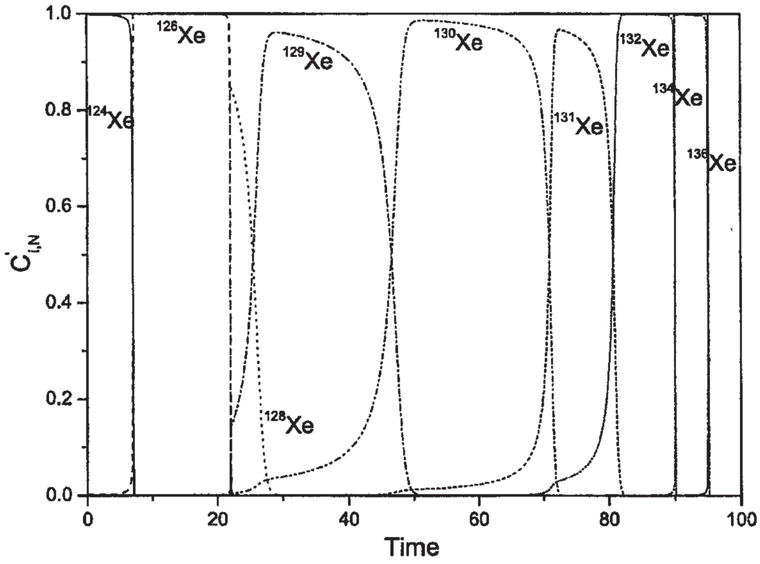


***Figure 6.*** The concentration vs. time in $P$ for $\gamma_0 = 1.4$, $P = 1$, $L'_n = 10{,}000$.

a component is narrowed. This, indeed, improves resolution but not thoroughly. A component suddenly appears but disappears gradually. The curves are still intertwined, no matter how large the separation factor is. In fact, roughly speaking, for the separation factor larger than 5, the concentration-time plots are not distinguishable from Fig. 4. Therefore, further improvement of separation performance cannot rely only on increasing the separation factor. Lengthening cascade is equivalent to increasing the separation factor, and so is not discussed further.

## The Effect of the Ratio $L'_n/P$

Now, we change the value of $L'_n$ and see what happens. Figures 5 and 6 present the concentration against time for $L'_n = 5$ and 10,000, respectively (See Fig. 3 for $L'_n = 10$). The improvement of resolution is clearly demonstrated when the values of $L'_n$ vary from 5 to 10,000. Two main observations need to be pointed out. First, in contrast to the effect of the separation factor, the appearance of a component occurs gradually but disappears quickly. This suggests that the combination of a larger $\gamma_0$ with a larger $L'_n$ may give us a very good resolution. Second, the larger $L'_n$ is, the better the resolution. But values of $L'_n$ greater than 1000 do not help to improve the resolution

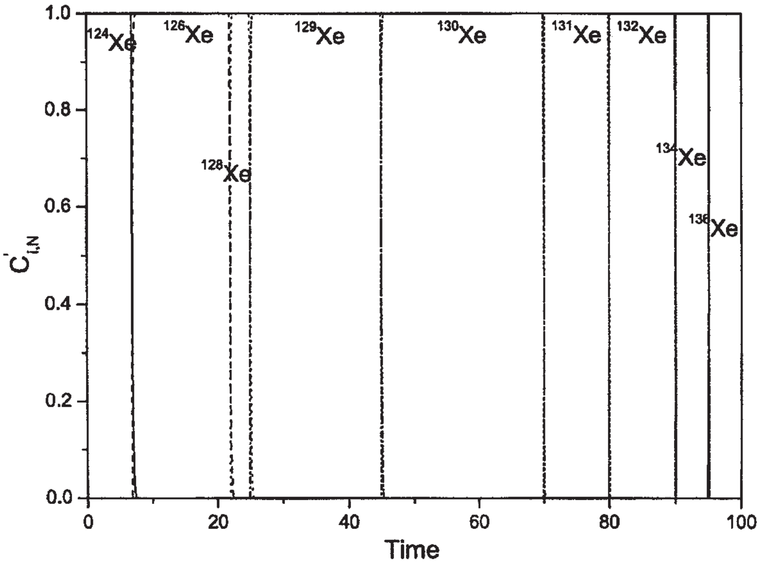


***Figure 7.*** The concentration vs. time for $\gamma_0 = 5$ and $L'_n/P = 1000$.

significantly. Remember that $P = 1$ here, so changing $L'_n$ is actually changing the ratio of $L'_n/P$. It is the ratio $L'_n/P$ that really affects the resolution. This is readily verified by numerical experiments: as long as the ratios $L'_n/P$ in two experiments are the same, the results are the same, with proper scaling of time. To show the enhancement of separation performance with larger $\gamma_0$ and $L'_n/P$, Fig. 7 presents the concentration vs. time for $\gamma_0 = 5$ and $L'_n/P = 1000$. One can see that the profiles of the concentrations of neighboring components have almost no overlapping and behave like square step functions that stand side by side one after another. In this case, the resolution is obviously very high and the components are almost completely separated. So in the withdrawal, here the product withdrawal, one obtains the lightest component, $^{124}$Xe, at the time interval from 0–7, then $^{126}$Xe from 7–22, etc., finally the heaviest component, $^{136}$Xe, from 95–100.

## Separation of a Middle Component by Using an NFSW Cascade

Suppose that the concentration of the $i$th component is to be enriched from concentration $C_i^F$ in the isotopic mixture to $C_i^P$ in the final product after $N_R$ separation runs of an NFSW cascade. Different from a conventional cascade, the concentration distribution is time dependent for nonstationary cascades. The $N_R$ runs may be split into two phases, as in the runs of the SW cascade[2], and illustrated in Fig. 8. The first phase consists of $\hat{N}_R$ runs, in which the material remaining in the reservoir $H_1''$ or $H_n'$ is collected for the next further separation, while in the second phase, the withdrawal $P_N$ or $W_1$ is collected for further separation. A separation run in the first phase is carried out as follows. Without loss of generality, suppose that the $r$th run in the first phase $H_1''$ is used to collect the material. When the concentration of the $i$th component in the reservoir $H_1''$ reaches maximum, denoted as

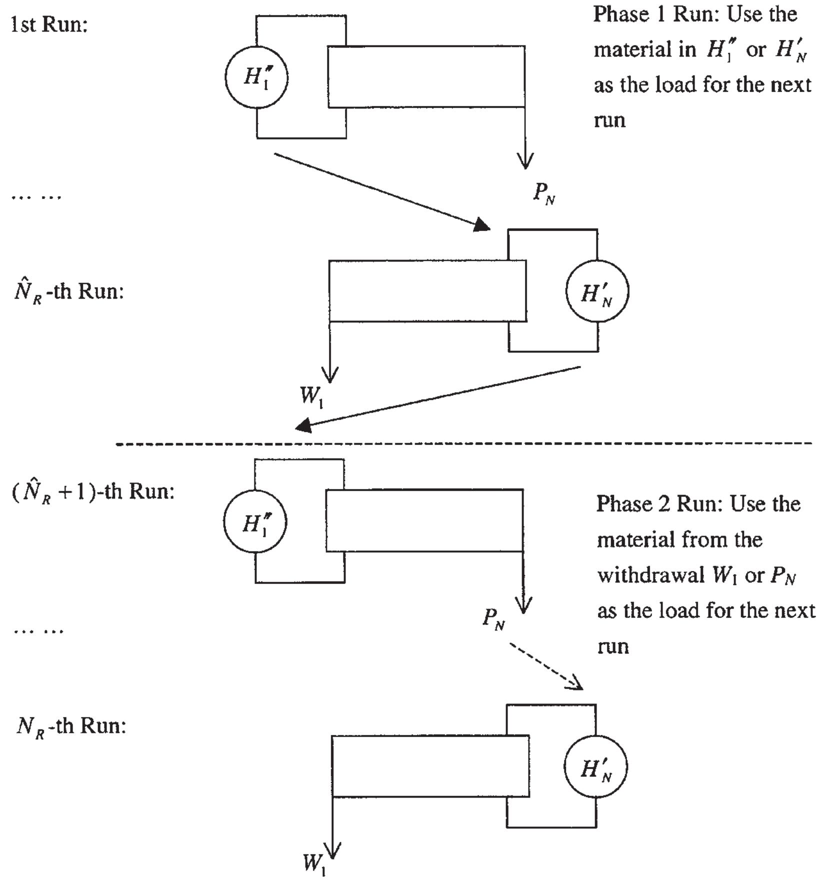


***Figure 8.*** The separation run queue for separating a middle component.

$C_{W,i}^{r}$, this run stops and the remaining material in $H_1''$ is collected. Let $T_1^r$ be the time spent, with the superscript $r$ indicating the run sequence. Then the time and the load needed for producing 1 unit mass of the material of concentration $C_{W,i}^{r}$ are, respectively,

$$T^r = \frac{T_1^r}{H_1''^{(0)} - PT_1^r}$$

$$M^r = \frac{H_1''}{H_1''^{(0)} - PT_1^r}$$

After a few runs of the first phase, the desired component has been enriched to some extend, and the loss of the component gradually becomes greater through the withdrawal. Then the operation should be switched to the second phase. At the withdrawal point, the concentration of the *i*th component vs. time may look like that in Fig. 9 in a separation run of the second phase. Now the withdrawal is collected when the concentration of the *i*th component in the withdrawal is larger than a given value $C_c^r$, referred to as the collection threshold concentration. The corresponding time for the

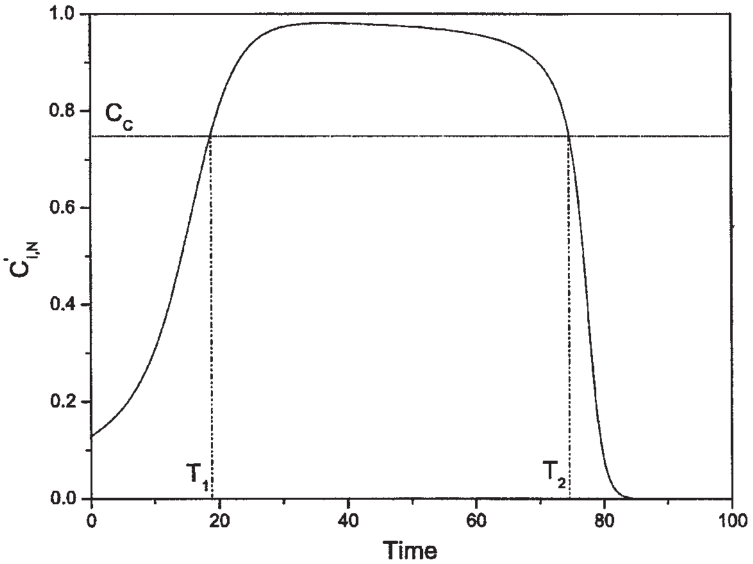


***Figure 9.*** The concentration of a component vs. time, the collection concentration threshold $C_c$ and the corresponding collection start time $T_1$ and stop time $T_2$.

collection is from $T_1^r$ to $T_2^r$. If the average concentration in the collected material is $\bar{C}_{P,i}^r$, then

$$\bar{C}_{P,i}^r = \frac{1}{T_2^r - T_1^r} \int_{T_1}^{T_2} C_{P,i}^r(t)dt$$

where $C_{P,i}^r(t)$ is the concentration of the $i$th component at the withdrawal of the $r$th run at time $t$. Clearly,

$$C_{F,i}^r = \begin{cases} C_i^F & r = 1 \\ \max(C_{w,i}^{r-1}) & r > 1 \text{ if Phase 1 run} \\ \bar{C}_{P,i}^{r-1} & r > 1 \text{ if Phase 2 run} \end{cases}$$

where $C_{F,i}^r$ is the concentration of the $i$th component in the initial load of the $r$th run. After the time $T_2^r$, the cascade operation is meaningless, because it does no useful separation of the required component. The amount of the product obtained for the $i$th component is $P(T_2^r - T_1^r)$. Therefore, the time to produce 1 unit mass of the separated material is

$$T^r = \frac{T_2^r}{P(T_2^r - T_1^r)}$$

and needs

$$M^r = \frac{H_1^{\prime\prime(0)}}{P(T_2^r - T_1^r)}$$

unit mass of load. It can be easily calculated that to produce 1 unit mass of the final product requires a total of

$$M = \prod_{r=1}^{N_R} M^r$$

unit mass of the isotopic mixture in the first run, and the total time of carrying out $N_R$ runs is

$$T = \sum_{r=1}^{N_R} \left( T^r \prod_{q=r+1}^{N_R} M^q \right) \tag{8}$$

The calculation of $T$ and $M$ for separation operations by using $H_n'$ or $W_1$ to collect the separated material can be carried out in the same way.

## Evaluation of the Separation Performance

In this paper, the separation performance is evaluated in terms of the material recovery $R$ and the operation time efficiency $E_t$ in producing 1 unit mass of the final product, without taking into account some practical factors such as operation cost. That is, the larger $R$ and $E_t$ are, the better the separation performance is.

The material recovery $R$ is the ratio of the total amount of the desired component in the product of the last run to the amount of the component in the total load of the first run and is calculated by the following equation:

$$R = \bar{C}_{P,i}^{N_R}/(C_i^F M)$$

The operation time efficiency $E_t$ is the ratio of the total time $T_0$ used by a sufficiently long cascade (cf. Zeng and Ying[2]) operating in a conventional way to the time $T$ used by the cascade to be evaluated to produce the same amount of product and evaluated by Eq. (8). It gives an idea about how much of the separation time of a cascade is effective compared with a sufficiently long cascade. Because a sufficiently long cascade can enrich any component, say the $i$th component, to any required concentration in at most two separation runs, $T_0$ is readily calculated by the following formula, provided that the feed rate is 1:

$$T_0 = \frac{C_i^P}{C_i^F}\left(1 + a\sum_{j=1}^{i} C_j^F\right) \tag{9}$$

if the product of the first run is used as the feed for the second run or

$$T_0 = \frac{C_i^P}{C_i^F}\left(1 + a\sum_{j=i}^{N_c} C_j^F\right) \tag{10}$$

if the waste of the first run is used as the feed for the second run. Here $a = 0$ if the $i$th component is an end component, namely $i = 1$ or $i = N_c$, and $a = 1$ otherwise. For achieving the shortest operation time, which of the two options is used depends on which sum, $\Sigma_{j=i}^{i} C_j^F$ or $\Sigma_{j=i}^{N_c} C_j^F$, is smaller.

## An Ideal Case

The ideal case of NFSW cascades is an NFSW cascade with very large $\gamma_0$ and $L_n'/P$, from the point of view of the material recovery and the operation time efficiency. It is easy to know from Fig. 7 that the material recovery $R = 1$, assuming that the concentration-time plots are strictly square step functions, and the operation time for producing 1 unit mass of the desired component is just $T = T_2/[P(T_2 - T_1)]$. For the separation of the $i$th component $T_1 = H_1''\Sigma_{j=1}^{i-1} C_j^F/P$ and $T_2 = H_1''\Sigma_{j=1}^{i} C_j^F/P$. Therefore,

$$T = \sum_{j=1}^{i} C_j^F/(PC_i^F)$$

Taking $P = 1$ and $C_i^p = 1$ gives

$$E_t = \frac{T_0}{T} = a + \frac{1}{\sum_{j=1}^{i} C_j^F} > 1$$

By using a definition of efficiency that is greater than 1 seems to be inappropriate, and, to modify this, it seems that an NFSW cascade with very large $\gamma_0$ and $L_n'/P$ should be the standard for the comparison of the operation time efficiency, and Eq. (15) should be used for $T_0$ instead of Eq. (9) or (10) in the calculation of the operation time efficiency. However, for the following simple reason, we still stick to the original definition, which was introduced in Zeng and Ying.[2] This ideal situation can be achieved only when the separation factor $\gamma_0$ and the ratio $L_n'/P$ are very large, like the case here, $\gamma_0 = 5$ and $L_n'/P = 1000$. This is very impractical. Because a stage with $\gamma_0 = 5$ is approximately equivalent to 5 stages with $\gamma_0 = 1.4$; a cascade of this kind is equivalent to about 130 cascades that can be considered to be sufficiently long. Recall that a conventional cascade with $\gamma_0 = 1.4$, $L_n' = 10$, and 41 stages can be thought of as being sufficiently long. (See Zeng and Ying.[9]) Therefore, $(N \times L'/P)_{\gamma 0=5}/N \times L'/P_{\gamma 0=1.4} = (N_{\gamma 0=5}/N_{\gamma 0=1.4}) \times [(L'/P)_{\gamma 0=5}/(L'/P)_{\gamma 0=1.4}] = (5 \times 11/41) \times (1000/10) \approx 134$). Choosing a very impractical case as the standard for comparison may not be sensible and may sometimes be misleading.

## NUMERICAL EXPERIMENT

Compared with the previous results[2], the separation of $^{130}$Xe in its natural isotopic mixture is chosen to be an example for the investigation of the NFSW cascade. The natural abundances of the nine Xenon components are given in Table 2. The concentration of $^{130}$Xe is required to be enriched from the concentration $C_5^F = 0.0408$ in the natural isotopic mixture to the concentration $C_5^P \geq 0.999$.

In the experiments, the cascade in the "ideal" case is not chosen, but instead rather practical cascades are considered. Without involving too many cases and for comparing with previous results, unless otherwise explicitly stated, the following specification is used: $\gamma_0 = 1.4$, $L_n' = 10$ $(n = 1,2, \ldots, N)$, $P = 1$, and $H_1''^{(0)} = 100$ (or $W = 1$ and $H_n' = 100$). Cascades of two cascade lengths, $N = 11$ and $N = 21$, are studied.

***Table 2.*** The natural concentrations of the nine Xenon components.

| Components | Concentration |
|---|---|
| $^{124}$Xe | 0.00095 |
| $^{126}$Xe | 0.0009 |
| $^{128}$Xe | 0.01917 |
| $^{129}$Xe | 0.2644 |
| $^{130}$Xe | 0.0408 |
| $^{131}$Xe | 0.2118 |
| $^{132}$Xe | 0.2689 |
| $^{134}$Xe | 0.1044 |
| $^{136}$Xe | 0.0887 |

## RESULTS AND DISCUSSION

In Table 3, the results for $T^r$, $M^r$, $T$, $M$, $R$, and $E_t$ are presented, which are obtained with only Phase 2 separation. The concentrations of $^{130}$Xe and its two neighboring components are also presented to give more details. The withdrawals at the product end and the waste end are alternately used for collection to achieve better separation. Before making any comparisons, the results obtained with both Phase 1 and Phase 2 separations used are presented in Table 4. In Phase 2, the threshold concentration $C_c^r$ is chosen to be the concentration of the desired component in the load. From the tables, two facts can be clearly noticed. First, increasing cascade length improves the separation performance significantly. Second, by using both Phase 1 and Phase 2 separations gives a much better performance than when using Phase 2 separation alone. For comparison, the results of $R$ and $E_t$ obtained by using conventional cascades and SW cascades (See Zeng and Ying[2] for more details) are given in Table 5. The three cases for a given SW cascade length correspond to three different operation time.

For the shorter cascade, both the SW cascade and the NFSW cascade are advantageous over the conventional cascade. For the longer cascade, the differences among the three cascades are lessened, but, in general, the SW cascade and the NFSW cascade are still superior to the conventional cascade. The NFSW cascade and the SW cascade are comparable with each other, except that in the longer cascade length case, the NFSW cascade shows extremely excellent operation time efficiency.

There are also other ways for further improving the material recovery, $R$, and the operation time efficiency, $E_t$. For example, the withdrawal rate $P$ (or $W$) and collection threshold concentration, $C_c^r$, may be the factors that should be taken into account for improvement. For a larger value of $C_c^r$ in the $r$th run

***Table 3.*** The results of $T^r$, $M^r$, $T$, $M$, $R$, $E_t$, and the concentrations of $^{130}$Xe and its neighboring components obtained with NFSW cascades operating only in Phase 1.

| Run no. | Collection | $C_4(^{129}\text{Xe})$ | $C_5(^{130}\text{Xe})$ | $C_6(^{131}\text{Xe})$ | $T^r$ | $M^r$ |
|---|---|---|---|---|---|---|
| Cascade length $N = 11$ | | | | | | |
| 1 | $P_N$ | 0.4346 | 0.0976 | 0.3692 | 1.34 | 0.354 |
| 2 | $W_1$ | 0.4180 | 0.2141 | 0.3450 | 1.84 | 0.374 |
| 3 | $P_N$ | 0.2906 | 0.3977 | 0.3032 | 1.60 | 0.429 |
| 4 | $W_1$ | 0.2371 | 0.6059 | 0.1568 | 1.46 | 0.503 |
| 5 | $P_N$ | 0.0775 | 0.7898 | 0.1325 | 1.27 | 0.604 |
| 6 | $W_1$ | 0.0682 | 0.8978 | 0.3401 | 1.16 | 0.723 |
| 7 | $P_N$ | 0.0142 | 0.9569 | 0.0288 | 1.12 | 0.806 |
| 8 | $W_1$ | 0.0126 | 0.9815 | 0.0059 | 1.08 | 0.864 |
| 9 | $P_N$ | 0.0024 | 0.9927 | 0.0049 | 1.09 | 0.877 |
| 10 | $W_1$ | 0.0021 | 0.9969 | 0.0010 | 1.08 | 0.894 |
| 11 | $P_N$ | 0.0004 | 0.9988 | 0.0008 | 1.09 | 0.889 |
| 12 | $W_1$ | 0.0004 | 0.9992 | 0.0004 | 1.00 | 0.999 |
| $M = 165.3$, $T = 154.7$, $R = 0.148$, $E_t = 0.220$ | | | | | | |
| Cascade length $N = 21$ | | | | | | |
| 1 | $P_N$ | 0.3956 | 0.1111 | 0.4253 | 1.44 | 0.327 |
| 2 | $W_1$ | 0.3590 | 0.2539 | 0.3781 | 1.73 | 0.381 |
| 3 | $P_N$ | 0.3026 | 0.4858 | 0.2077 | 1.57 | 0.438 |
| 4 | $W_1$ | 0.0691 | 0.7395 | 0.1914 | 1.31 | 0.524 |
| 5 | $P_N$ | 0.0904 | 0.8947 | 0.0149 | 1.00 | 0.762 |
| 6 | $W_1$ | 0.0030 | 0.9800 | 0.0170 | 1.00 | 0.876 |
| 7 | $P_N$ | 0.0031 | 0.9963 | 0.0005 | 1.00 | 0.970 |
| 8 | $W_1$ | 0.0003 | 0.9992 | 0.0005 | 1.00 | 0.996 |
| $M = 54.2$, $T = 48.1$, $R = 0.452$, $E_t = 0.675$ | | | | | | |

(in Phase 2), $\bar{C}^r_{P,i}$ would be larger, and so the total number of runs $N_R$ would be smaller. This means that the concentration of the required component can be achieved with a fewer number of runs, comparing the case with a smaller value of $C^r_c$. However, the larger $C^r_c$ is, the smaller $R^r$ is. For $E^r_t$, there may exist a maximum as $C^r_c$ increases. Actually, it is $E_t$ and $R$ of the whole separation process that are the parameters one really cares about other than $E^r_t$ and $R^r$ of individual runs. The $E_t$ and $R$ are a function of $P^r$ (or $W^r$, but we ignore mentioning it here for simplicity of explanation) and $C^r_c$ of each run, that is, $E_t = E_t$ $(P^1, \ldots, P^{N_R}, C^1_c, \ldots, C^{N_R}_c)$, and $R = R(P^1, \ldots, P^{N_R}, C^1_c, \ldots, C^{N_R}_c)$. The best result can be obtained by optimizing some kind of couplings of $E_t$ and $R$ with respect to $P^r$ and $C^r_c$ (r = 1,2,. . ., $N_R$). Considering $E_t$ and $R$ in coupled ways is necessary, because optimizing them separately leads to useless results. An

***Table 4.*** The results of $T^r$, $M^r$, $T$, $M$, $R$, $E_t$, and the concentrations of $^{130}$Xe and its neighboring components obtained with NFSW cascades operating in both Phase 1 and Phase 2.

| Run no. | | Collection | $C_4(^{129}\text{Xe})$ | $C_5(^{130}\text{Xe})$ | $C_6(^{131}\text{Xe})$ | $T^r$ | $M^r$ |
|---|---|---|---|---|---|---|---|
| Cascade length $N = 11$ | | | | | | | |
| Phase 1 | 1 | $H'_N$ | 0.6950 | 0.0960 | 0.1198 | 1.398 | 2.398 |
| | 2 | $H''_1$ | 0.1609 | 0.2703 | 0.5235 | 2.359 | 3.359 |
| | 3 | $H'_N$ | 0.3910 | 0.4944 | 0.1132 | 1.233 | 2.233 |
| | 4 | $H''_1$ | 0.0812 | 0.6844 | 0.2330 | 0.885 | 1.885 |
| Phase 2 | 5 | $P_N$ | 0.1083 | 0.8311 | 0.0606 | 1.000 | 1.336 |
| | 6 | $W_1$ | 0.0250 | 0.9252 | 0.0498 | 1.162 | 1.342 |
| | 7 | $P_N$ | 0.0223 | 0.9671 | 0.0106 | 1.095 | 1.194 |
| | 8 | $W_1$ | 0.0043 | 0.9868 | 0.0089 | 1.094 | 1.156 |
| | 9 | $P_N$ | 0.0038 | 0.9944 | 0.0016 | 1.077 | 1.124 |
| | 10 | $W_1$ | 0.0007 | 0.9978 | 0.0015 | 1.088 | 1.126 |
| | 11 | $P_N$ | 0.0006 | 0.9991 | 0.0003 | 1.075 | 1.114 |
| | | $M = 118.1$, $T = 126.5$, $R = 0.207$, $E_t = 0.257$ | | | | | |
| Cascade length $N = 21$ | | | | | | | |
| Phase 1 | 1 | $H'_N$ | 0.7125 | 0.1069 | 0.1157 | 1.451 | 2.451 |
| | 2 | $H''_1$ | 0.1428 | 0.3731 | 0.4515 | 2.509 | 3.509 |
| | 3 | $H'_N$ | 0.2923 | 0.6181 | 0.0882 | 0.862 | 1.862 |
| Phase 2 | 4 | $W_1$ | 0.0338 | 0.8339 | 1.1310 | 1.000 | 1.493 |
| | 5 | $P_N$ | 0.0410 | 0.9523 | 0.0068 | 1.000 | 1.215 |
| | 6 | $W_1$ | 0.0012 | 0.9916 | 0.0072 | 1.000 | 1.064 |
| | 7 | $P_N$ | 0.0012 | 0.9986 | 0.0002 | 1.000 | 1.016 |
| | 8 | $W_1$ | 0.0006 | 0.9992 | 0.0002 | 1.000 | 1.001 |
| | | $M = 31.4$, $T = 34.9$, $R = 0.779$, $E_t = 0.931$ | | | | | |

example is that, if only $R$ is optimized without taking into account $E_t$, then a large $R$ (close to 1) can be obtained by having $P^r$ very small and $C_c^r$ as large as possible. This gives very unacceptably small $E_t$. Therefore, the coupled optimization appears to be somewhat complicated. To simplify the study, only for the purpose of demonstrating the effects of $P^r$ and $C_c^r$, $P^r$ and $C_c^r$ are chosen in the following way. For a given cascade length, $P^r$ is the same for all runs. An easy to choose option for $C_c^r$ is $C_c^r = C_{F,i}^r$ for all runs except the last run, in which $C_c^r$ should be chosen such that $\bar{C}_{P,i}^{N_R}$ is equal to the required concentration. The results for $R$ and $E_t$ are presented in Figs. 10 and 11 for $N = 11$ and $N = 21$, respectively. The material recovery always decreases monotonically as $P$ increases, but the operation time efficiency has a maximum.

*Table 5.* The material recovery $R$ and the operation time efficiency $E_t$ obtained with conventional cascades and SW cascades.

| Cascade | Cascade length $N$ | Run case | No. runs $N_R$ | $R$ | $E_t$ |
|---|---|---|---|---|---|
| Conventional | 11 | | 13 | 0.104 | 0.082 |
| | 21 | | 7 | 0.743 | 0.635 |
| SW | 11 | 1 | 11 | 0.123 | 0.249 |
| | | 2 | 12 | 0.240 | 0.266 |
| | | 3 | 13 | 0.308 | 0.331 |
| | 21 | 1 | 7 | 0.615 | 0.669 |
| | | 2 | 9 | 0.779 | 0.781 |
| | | 3 | 11 | 0.843 | 0.787 |

This does not mean that $P$ should take the value when $E_t$ is maximum. What a suitable value of $P$ is varies from case to case; for example, if the material is very expensive, then $R$ should take precedence over $E_t$.

To demonstrate further how the choice of $C_c^r$ affects $R$ and $E_t$, a few more cases of different cascade lengths are investigated by taking $C_c^r = C_{F,i}^r$ and $C_c^r = (C_{F,i}^r + C_{F,i}^{r-1})/2$. The results are given in Table 6 and also are plotted in Figs. 12 and 13 as well. The $P_{max}$ correspondes to the

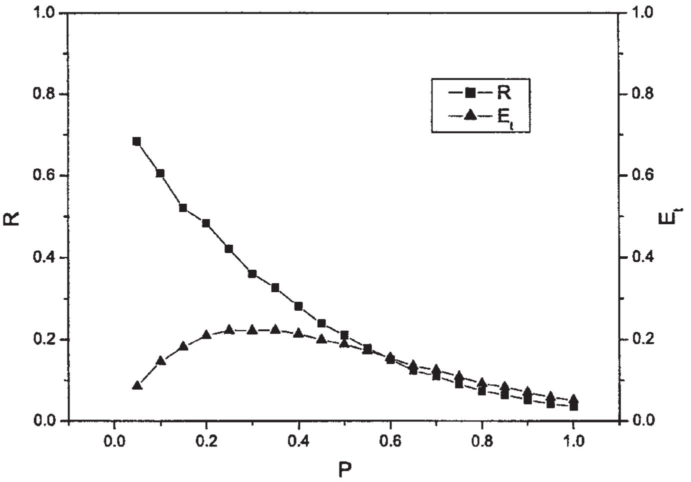


*Figure 10.* $R$ and $E_t$ obtained by taking $C_c^r = C_{F,i}^r$ for $N = 11$.

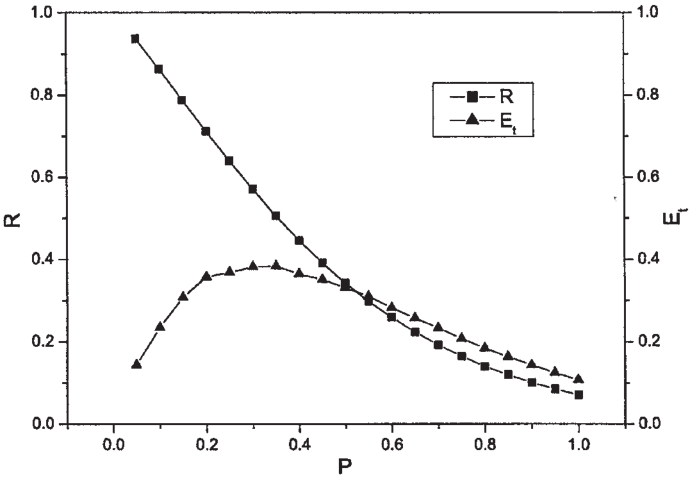


***Figure 11.*** $R$ and $E_t$ obtained by taking $C_c^r = C_{F,i}^r$ for $N = 21$.

***Table 6.*** The effects of the choice of the collection concentration threshold $C_c^r$ on the material recovery rate $R$ and the operation time efficiency $E_t$.

| $N$ | $P_{\max}$ | $N_R$ | $C_{P,5}^{N_R}$ | $R$ | $E_t$ |
|---|---|---|---|---|---|
| $C_c^r = C_r^{F,5}$ | | | | | |
| 6 | 0.28 | 12 | 0.999 | 0.0486 | 0.0211 |
| 11 | 0.34 | 7 | 0.999 | 0.335 | 0.223 |
| 16 | 0.31 | 5 | 0.999 | 0.512 | 0.349 |
| 21 | 0.33 | 5 | 0.999 | 0.531 | 0.385 |
| 26 | 0.33 | 5 | 0.999 | 0.543 | 0.394 |
| 31 | 0.34 | 5 | 0.999 | 0.533 | 0.396 |
| 36 | 0.34 | 5 | 0.999 | 0.534 | 0.397 |
| 41 | 0.37 | 5 | 0.999 | 0.498 | 0.395 |
| $C_c^r = (C_{F,5}^{r-1} + C_{F,5}^r)/2$ | | | | | |
| 6 | 0.27 | 14 | 0.999 | 0.0786 | 0.0294 |
| 11 | 0.31 | 8 | 0.999 | 0.405 | 0.230 |
| 16 | 0.35 | 7 | 0.999 | 0.493 | 0.332 |
| 21 | 0.36 | 7 | 0.999 | 0.519 | 0.360 |
| 26 | 0.40 | 7 | 0.999 | 0.482 | 0.364 |
| 31 | 0.42 | 7 | 0.999 | 0.417 | 0.327 |
| 36 | 0.42 | 7 | 0.999 | 0.384 | 0.302 |
| 41 | 0.42 | 7 | 0.999 | 0.383 | 0.302 |

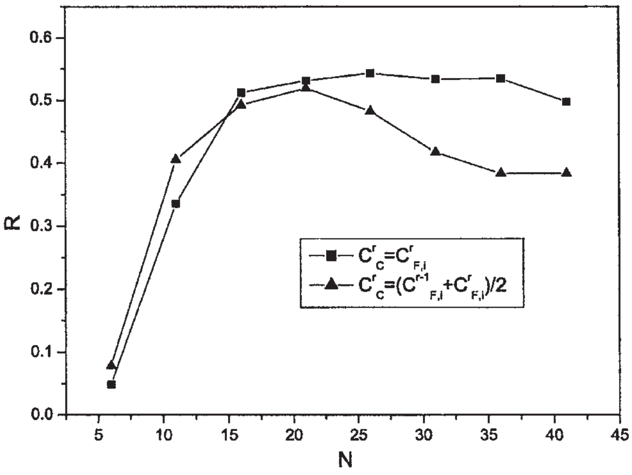


*Figure 12.* R and $E_t$ for different cascade length with $C_c^r = C_{F,i}^r$.

values where $E_t$ reaches maxima. It is understandable that with the increase of cascade length, the separation performance is enhanced and so are $R$ and $E_t$. However, the results show that $R$ and $E_t$ are not always improved; the values of $R$ and $E_t$ for $N = 41$ are somewhat smaller than those for $N = 31$ in the first choice of $C_c^r$ and are apparently smaller than those even for

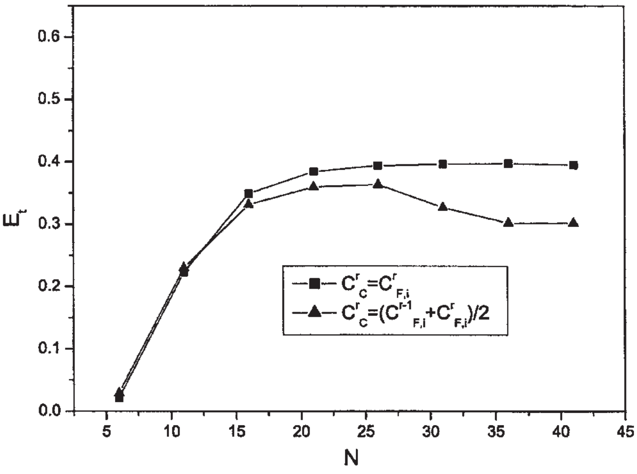


*Figure 13.* R and $E_t$ for different cascade lengths with $C_c^r = (C_{F,i}^r + C_{F,i}^{r-1})/2$.

$N = 16$ in the second choice. This indicates that a choice of $C_c^r$ for a cascade of certain length may not be suitable for another cascade of different length. Therefore, $C_c^r$ should be determined according to the actual situation. In the example here, the first choice is better than the second.

## CONCLUSIONS

A nonstationary gas centrifuge cascade with no feed and only a single withdrawal (the NFSW cascade) is investigated for the separation of middle components in isotopic mixtures and is compared with the conventional cascade and another nonstationary cascade (the SW cascade) with respect to the material recovery and the operation time efficiency in the separation of $^{130}$Xe as an example.

By using a very large separation factor $\gamma_0$ or, equivalently, a long cascade alone cannot produce the largest separation performance, namely, the ideal case. It can be obtained when the ratio $L_n'/P$ (or $L_n'/W$) is large at the same time with a large $\gamma_0$. However, cascades with very large $\gamma_0$ and also very large $L_n'/P$ (or $L_n'/W$) as well may not be practical.

In a practical case with $\gamma_0 = 1.4$, $L_n' = 10$, and $P = 1$ (or $W = 1$), two NFSW cascades of lengths $N = 11$ and $N = 21$ are studied. They are advantageous over the corresponding two conventional cascades and comparable with the corresponding two SW cascades of the same lengths. For $N = 21$, the operation time efficiency of the NFSW cascade is the best of all cases. Therefore, of the three types of cascades, the NFSW cascade may be the choice for the separation of middle components.

The separation performance can be further improved by means of optimization with respect to the withdrawal rate $P$ (or $W$). For a fixed $L_n'$, the material recovery is monotonically decreased as increasing $P$, but the operation time efficiency has a maximum. This suggests that one has to make a compromise between the two factors. The choice of the collection threshold concentration also affects the performance. Here two choices were investigated, the first was taking the threshold concentration to be the concentration of the desired component in the load, and the second was the average of the concentrations of the desired component in the loads of the current run and the previous run. The first choice is more appropriate than the second one, because the second leads to the obvious decrease of both the material recovery and the operation time efficiency with the increase of cascade length. But neither of the two choices may be the best, which should be determined according to the given cascade.

To summarize, nonstationary cascades have shown advantages over conventional cascades in separating middle components in our preliminary investigation here. We think that the technique of using nonstationary cascades is worth further exploration.

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